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Research Paper

Use of the wavelet transforms for time of flight estimation of overlapping ultrasonic signals in thin coatings

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ABSTRACT

Echoes detection and Time Of Flight (TOF) estimation is a challenging task in ultrasonic signal processing, especially in the case of overlapping echoes. Among existing methods, the wavelet transform method is the most used to process such signals. In this study, two methods based on the wavelet transform were investigated in order to yield the most accurate estimation of the TOF of overlapping echoes. The first one is the continuous wavelet transform based on Scale-Averaged Power (SAP), while the second method is the Discrete Stationary Wavelet Transform (DSWT). Both methods were applied for simulating Gaussian signals for two cases: partially and closely spaced overlapping echoes. To select a suitable mother wavelet, as well as the decomposition level for each method, several numerical tests have been carried out on the simulated signals. The performance was evaluated using the Mean Square Error (MSE) between the estimated value of the TOF and the actual value. Simulation results showed that both methods estimate the TOF with a reasonable error for the case of partially overlapping echoes. In the case of closely spaced overlapping echoes, the accuracy and the limit of the wavelet transform methods were obtained. Experimental validation was performed on real signals taken from thermally coated samples. A remarkable agreement was observed between simulations and experiments.

Key words: Time of flight, ultrasonic signals, overlapping echoes, SAP, DSWT, Thin coating.

S. Laddada[1,2*], S. Lemlikchi[1], M. O. Si-Chaib[2], H. Djelouah[3] and A. Yahiaoui[4]

[1]Centre de Développement des Technologies Avancées, Baba-Hassen, 16303 Algérie.
[2]Solid Mechanics and Systems Laboratory (LMSS), University M'hamed Bougara Boumerdes, Algeria.
[3]Laboratoire de Physique des Matériaux, USTHB, Algérie.
[4]Laboratory of Dynamics Motors and Vibroacoustics 800Lgts boumerdes UMBB, Algeria.

*Corresponding author. E-Mail: laddadas@gmail.com.

INTRODUCTION

The accurate estimation of the ultrasonic time of flight is essential in Non-Destructive Testing and Evaluation (NDT&E). The TOF offers a capability of a rigorous coating inspection by evaluating the thickness and the presence of defects, as well as their size and shape. However, measuring the thin coating thickness can be difficult due to the overlapping, interfering and sharp transient characteristics of the reflected signals. The accuracy of the TOF estimation decreases critically and requires other advanced processing methods. Several time-frequency techniques based on the TOF estimation have been suggested so far for this resolution (Haase et al., 2003; Carmona et al., 1998). The main advantage of the time-frequency analysis is its capability to describe how the signal spectrum changes with the time, even if the spectral countenance is non-stationary. The Wavelet Transform (WT) can be considered as a powerful tool to describe the time-frequency evolution of non-stationary signals (Yahiaoui et al., 2016; Angrisani et al., 1997). It provides a good compromise between the time and frequency resolutions (Mallat, 1999; Daubechies, 1992). This was achieved by decomposing the original signal into a sum of elementary contributions called wavelets. The wavelets are generated by dilations and translations, as well as by the shifts of one single function (basic function) called the mother wavelet (Vetterli et al., 1992). Indeed, the wavelet transforms can be classified into two classes, namely the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT).

The CWT may be defined as a mapping dependent on the specification of an auxiliary function called the analyzing wavelet (Teolis et al., 1998). It computes the inner product of the signal with the translated and dilated versions of the analyzing wavelet. The CWT allows the estimation of the TOF using the envelope of Scale-Averaged Power (SAP) signal. It has already been used in various fields such as the tissue characterization (Georgiou et al., 2001), and structural damage identification (Su et al., 2009; Hosseinabadi et al., 2014).

The discrete wavelet transform is an implementation of the wavelet transform using a discrete set of the wavelet scales and translations. It has been used to estimate the TOF in many applications (Angrisani et al., 1997; Ingaroca et al., 2011). This method offers a low computational complexity due to the implementation of the wavelet filter banks. However, it suffers from a serious drawback since it is not a time-invariant transform and is a critically sampled transform. Moreover, the decimation makes it non-redundant. It was observed that the inherent deficiency of the DWT is the energy leakages caused by reduction of spatial dimensions of the original signal (Peng et al., 2009). To overcome the above limitations of the DWT, the Discrete Stationary Wavelet Transform (DSWT) was introduced Pesquet et al. (1996). The lack of the decimation ensures the translation invariance during the analysis. Furthermore, the time-invariance property of the DSWT is particularly important in many applications such as detection and parameter estimation of signals with unknown arrival time (Pesquet et al., 1996; Zhong et al., 2007). The envelope of the DSWT detail coefficients was exploited to estimate the TOF of ultrasonic echoes using the analytic signal. Different coating thicknesses of drug tablets coated with Kollicoat have been estimated using this envelope (Bikiaris et al., 2012).

The above reviewed studies have applied the wavelet transform techniques to evaluate the TOF that is important for estimating the coating thickness and identifying defects in materials. The authors have usually carried out experiments or simulation studies to get accurate results. However, experimental studies are very costly and time consuming especially when the method is limited in analyzing overlapping echoes. Identifying the accuracy and limits of the wavelet transform methods are really needed to avoid risks that are associated with experiments.

The present study proposes the use of numerical simulations to select the suitable mother wavelet and the appropriate number of decomposition levels that should be applied to analyze overlapping signals for estimating the thicknesses of coatings. To do so, two methods based on the envelope of wavelet transform, have been considered namely, the CWT based on Scale- Averaged Power (SAP) and the DSWT. In the DSWT, the signal is successively decomposed into approximation and detail coefficients. Only the detail coefficients are considered since they contain the useful information. The analytic signal is then used for making the envelope of the selected detail coefficient. To select the suitable mother wavelet and the appropriate number of decomposition levels for each method, several numerical tests have been carried out on both simulated and experimental signals. The simulated signals consist of two Gaussian echoes with different TOFs. Two cases were investigated: partially overlapping echoes and closely spaced overlapping echoes. The experimental signals were backscattered from known thickness FSX 414 coating deposited on stainless steel (SS) substrate. The performance of each method was evaluated using the mean square error (MSE) criterion and the accuracy of TOF estimation was established.

WAVELET TRANSFORM METHODS

SAP method

expression of the Continuous Wavelet Transform (CWT) for one-dimensional real signal is defined as (Rioul et al., 1991):

$$\mathrm{CWT}(s,\tau) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} y(t)\psi_0{}^{*}\left(\frac{t-\tau}{s}\right) dt \quad (1)$$

where ψ_0 is the mother wavelet, $*$ denotes the conjugate, $y(t)$ is the signal to be transformed, s and τ are the scale and translation parameters, respectively.

The CWT is defined by the inner product of the function and the basis wavelet. For the discrete realization of the CWT, we deal with sampled version of the real signal. The signal is sampled at a sampling rate f_s and recorded in the vector y (n), described by N sampling points, (n = 1, 2, ..., N). The CWT of the discrete signal y (n) is defined as the convolution of y (n) with a scaled and translated version of $\psi_0(n)$ (Georgiou et al., 2001):

$$\mathrm{CWT}(s,n) = \sum_{n'=0} y(n')\psi^{*}\left[\frac{(n'-n)}{sf_s}\right] \quad (2)$$

The scale averaged wavelet power (SAP) is defined as (Georgiou et al., 2001; Su et al., 2009; Hosseinabadi et al., 2014):

$$\mathrm{SAP}(n) = \frac{1}{M}\sum_{j=1}^{M}\left|\mathrm{CWT}(s_j,n)\right|^2 \quad (3)$$

where s_j denotes the scale and M is the largest scale during the CWT.

DSWT method

The DWT is the first discrete implementation of the wavelet

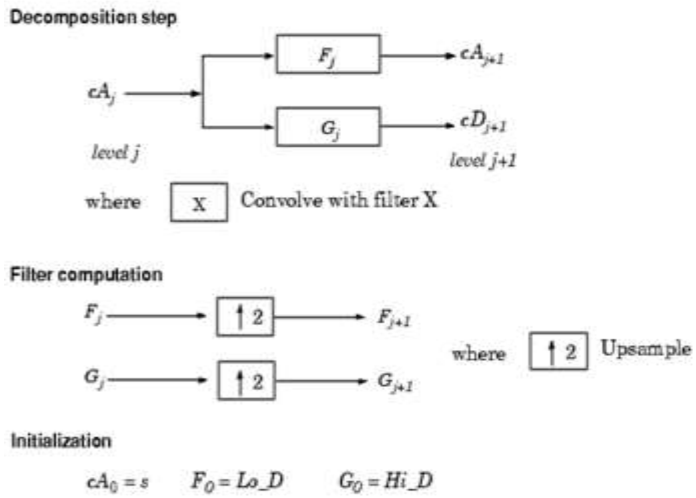

Figure 1: Block diagram of DSWT [18].

transform (Mallat, 1999; Daubechies, 1992). The Multi-Resolution Analysis (MRA) is the design method of a most practically relevant discrete wavelet transform and the justification for the algorithm of the fast wavelet transform (FWT) (Mallat, 1999). The DWT consists of filtering the input signal with a low pass filter and a high pass filter leading to two sub-bands called respectively approximations and details, followed by a decimation factor of both.

The decomposing formula of the DWT y(t) could be expressed as a function of the scaling function $\varphi_{j,k}(t)$ and the wavelet function $\psi_{j,k}(t)$ as:

$$y(t) = \sum_{k} a_{j_0 k} \varphi_{j_0,k}(t) + \sum_{j=j_0}^{\infty} \sum_{k} d_{jk}\ \psi_{j,k}(t) \tag{4}$$

$$\varphi_{j,k}(t) = \frac{1}{\sqrt{2^j}}\ \varphi\left(\frac{t-2^j\ k}{2^j}\right) \tag{5}$$

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}} \psi\left(\frac{t-2^j k}{2^j}\right) \tag{6}$$

where the subscribes j_0, j and k represent the greatest scale in the decomposition, the dilatation and the translation parameters respectively; $a_{jk} = \langle y, \varphi_{j,k}(t)\rangle$ are the approximation coefficients related to low frequency signal components; $d_{jk} = \langle y,\ \psi_{j,k}(t)\rangle$ are the detail coefficients representing the high frequency components.

Since the DWT coefficients are not time invariant, the DWT may lead to a loss of useful information at high frequency. Then, the DSWT is designed as an extension method of the DWT.

The discrete stationary wavelet transform DSWT is shown in Figure 1. It is the undecimated version of the DWT (Daubechies, 1992). The step $(j+1)$ convolves the approximation coefficients at level j with up-sampled versions of the appropriate original filters to produce the approximation and detail coefficients at level $(j+1)$ (Pesquet et al., 1996; Misiti et al. (2015).

For getting better performances for estimating the TOF, the analytic signal is used for extracting the envelope of the DSWT detail coefficients. An analytic signal is a complex signal where the real part is the original signal and the imaginary part is its Hilbert transform (Huang, 2014). For an arbitrary signal y(t), the analytic signal z(t) is defined as:

$$z(t) = y(t) + i\ \tilde{y}(t) \tag{7}$$

where i is the imaginary unit, and $\tilde{y}(t)$ is the Hilbert transform. The analytic signal z(t)can also be expressed as:

$$z(t) = |z(t)| e^{i\theta(t)} \tag{8}$$

where |z(t)|is the envelope of the analytic signal and θ(t) is its phase (Huang, 2014).

To extract the envelope of the DSWT, the detail coefficient at level j(DSWT (j; t)) is made equal to y(t). Once the two strongest peaks of |z(t)| are found, the TOF could be estimated (Bikiaris et al., 2012).

RESULTS AND DISCUSSION

Simulation part

In this study, we use the Gaussian echo model to simulate the ultrasonic signals which can be modeled as (Demirli et al., 2001):

Table 1: Simulated signals.

Parameter	Arrival time (μs)	
	Partially overlapping	Closely spaced overlapping
First echo	0.9	0.9
Second echo	1.02	0.99
TOF	0.12	0.09

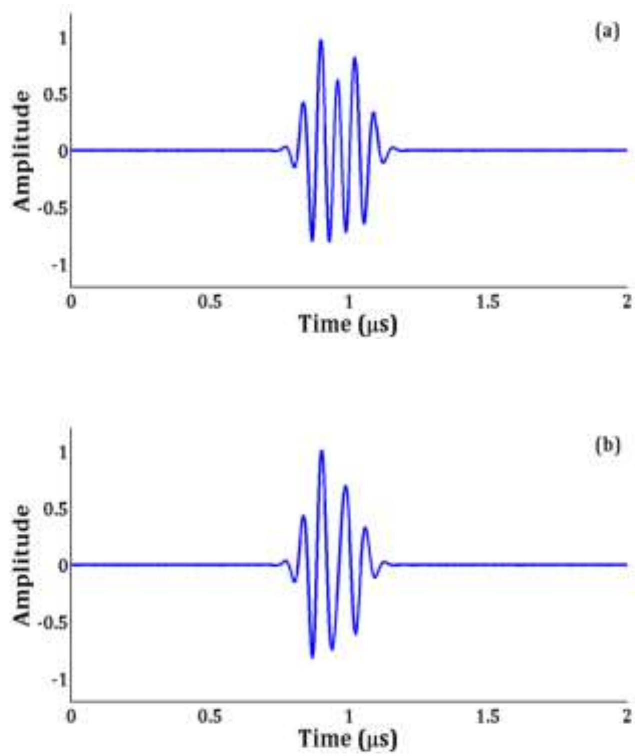

Figure 2: Simulated signals. (a) Partially overlapping echoes with TOF=0.12μs. (b) Closely spaced overlapping echoes with TOF=0.09μs.

$$\boldsymbol{x}(\mathrm{t}) = \beta e^{-\alpha(t-\tau)^2} \cos(2\pi f_c(t-\tau)+\emptyset)+\boldsymbol{\nu}(t) \quad (9)$$

where $\boldsymbol{x}$ is the real signal and $\boldsymbol{\nu}$is a White Gaussian Noise (WGN) sequence. β, α, τ, f_c, and $\emptyset$ are the amplitude, the bandwidth factor, the arrival time, the center frequency and the phase, respectively. The studied signals are the sum of two Gaussian echoes (Equation 9) simulated for two practical cases namely, partially overlapping echoes and closely spaced overlapping echoes. In each case, the first echo parameters are $\beta = 1$, $\alpha = 200(\mathrm{MHz})^2, f_c = 15$ MHz, and $\emptyset = 0$ rad and the second echo parameters are $\beta = 0.8, \alpha = 180\ (\mathrm{MHz})^2$, $f_c = 14$ MHz, and $\emptyset = 0$ rad. The signals were simulated with a sampling frequency of 500 MHz and WGN of 60 dB signal-to-noise ratio (SNR). To create the two situations, the arrival time of the first echo was set to $\tau_1 = 0.9$ μs, whereas the arrival time of the second echo was varied. The arrival time and the TOF of the first and the second echoes in each case are reported in Table 1. Figure 2 shows the simulated signals. Figure 2a shows the case of the partially overlapping echoes with a TOF equal to 0.12 μs. Figure 2b shows the case of the closely spaced overlapping echoes with a TOF equal to 0.09 μs.

The SAP and the DSWT methods have been applied on simulated signals. We note that several wavelet families results are eliminated from tables because some wavelets provided high errors and others could not detect the two strongest peaks of the processed signals.

Table 2 shows the main TOF estimation results obtained using the SAP method. This table contains the analyzing wavelet, the actual and the estimated TOF, the MSE and the scale. The wavelets included in this study, belong to the following families: Daubechies (db1 until

Table 2: Simulation results: SAP method.

Wavelet	Estimated TOF (µs)	MSE (%)	Scale
Actual TOF= 0.12 µs			
db1	0.119	0.83	12
db10, sym9	0.119	0.83	8
sym1	0.119	0.83	12.15
Actual TOF= 0.09 µs			
db7, db9	0.12	33.33	9
db1	0.12	33.33	5

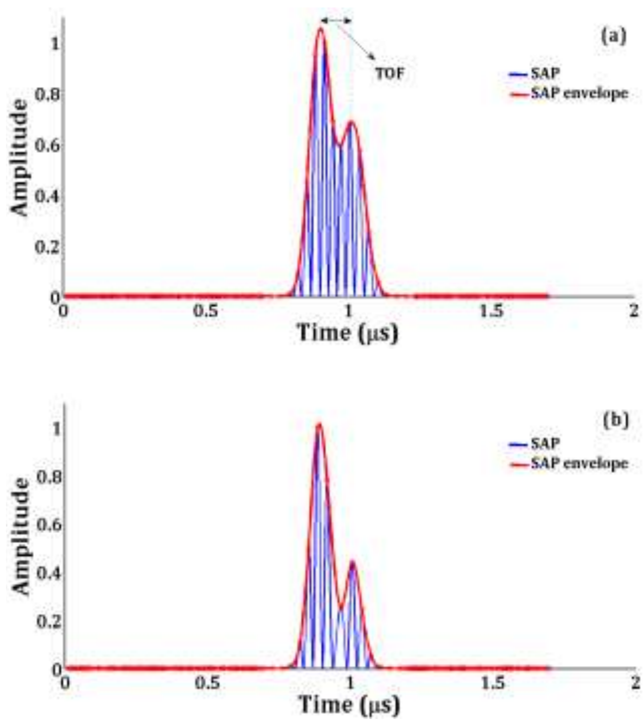

Figure 3: SAP envelope. (a) Partially overlapping echoes using db1 wavelet for the scale 12. (b)Closely spaced overlapping echoes using db7 wavelet for the scale 9.

db10), Symlet (sym1 until sym10), and Coiflet (coif1until coif5).

In the first case, for partially overlapping echoes where the actual TOF is equal to 0.12 µs, the best results were obtained using db1 wavelet for the scale 12, db10 and sym9 wavelet for the scale 8 and sym1 wavelet for the scale 12 and 15. The obtained TOF and the MSE were 0.119 µs and 0.83%, respectively. Finally, in the case of closely spaced overlapping echoes where the actual TOF is equal to 0.09 µs, the obtained TOF and the MSE were respectively 0.12 µs and 33.33%. These results were obtained using db7 and db9 wavelets for the scale 9 and db1 wavelet for the scale 5.

Figure 3 shows the SAP envelope for the two cases; the blue line represents the SAP and the red line represents the SAP envelope. Figure 3a shows the SAP and its envelope obtained using db1 wavelet for the scale 12 in the case of partially overlapping echoes. The two peaks in the envelope plot correspond to arrival times of 0.90 and 1.019 µs. This corresponds to a TOF of 0.119 µs.

Figure 3b shows the SAP and its envelope obtained using db7 wavelet for the scale 9 in the case of closely spaced overlapping echoes. The two strongest peaks of the SAP envelope correspond to arrival times of 0.92 and 1.04 µs. This corresponds to a TOF of 0.12 µs.

Table 3 shows the main TOF estimation results obtained by the use of the DSWT method. The analyzing wavelet, the actual and estimated TOF, the MSE and the scale are reported in this Table. The wavelets included in this study

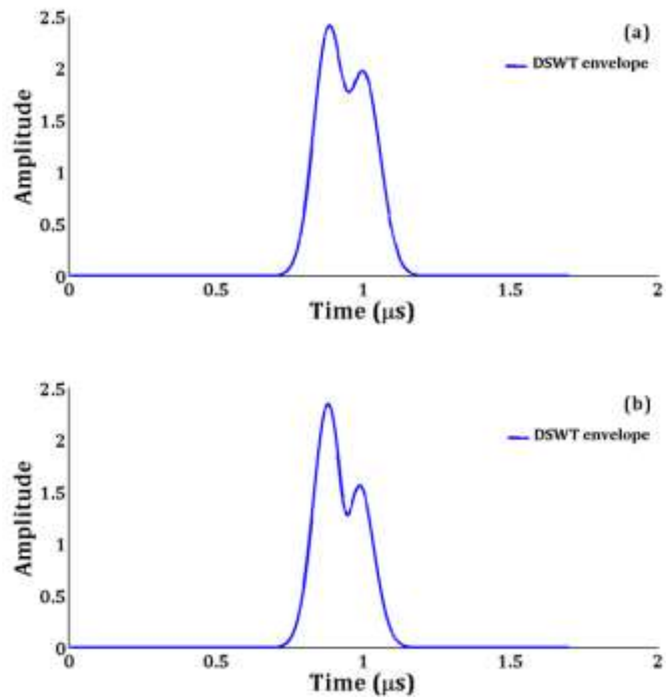

Figure 4: DSWT envelope. (a) Partially overlapping echoes using sym1 wavelet for the level 4.(b) Closely spaced overlapping echoes using coif1 wavelet for the level 4.

Table 3: Simulation results: DSWT method.

Wavelet	Estimated TOF (µs)	MSE (%)	Level
Actual TOF= 0.12 µs			
db1, sym1	0.119	0.83	4
db2, sym2	0.119	0.83	5
Actual TOF= 0.09 µs			
db1,sym1, coif1	0.11	22.22	4

belong to the following families: Daubechies (db1 until db10), Symlet (sym1 until sym10), and Coiflet (coif1until coif5).

As can be seen, the best estimate of the TOF in the first case was equal to 0.119 µs by the use of db1, sym1 wavelet for the level 4 and db2, sym2 wavelet for the level 5. The corresponding MSE was equal to 0.83%. The obtained TOF and the MSE were 0.11 µs and 22.22%, respectively in the most critical case closely spaced overlapping echoes. These results were obtained using db1, sym1 and coif1 wavelets for the level 4.

Figure 4 shows the DSWT envelope for the two cases. Figure 4a shows the DSWT envelope of partially overlapping echoes using sym1 wavelet for the level 4. The two peaks in the envelope plot correspond to arrival times of 0.89 and 1.009 µs. This corresponds to a TOF of 0.119 µs. Figure 4b shows the DSWT envelope obtained using coif1 wavelet for the level 4 in the case of closely spaced overlapping echoes. The two strongest peaks of the envelope are located at 0.88 and 0.99 µs. This corresponds to a TOF of 0.11 µs.

Experimental validation

To highlight the effectiveness of the simulated results, an experimental validation was carried out on thermally sprayed stainless steel (SS) samples. The coats were made by FSX414 alloy with two different thicknesses of 280 and 180 µm. This experience was based on the pulse echo technique generated by a transducer with a 15 MHz working frequency.

The backscattered signal was recorded on digital oscilloscope with 0.5 GHz sampling frequency. The

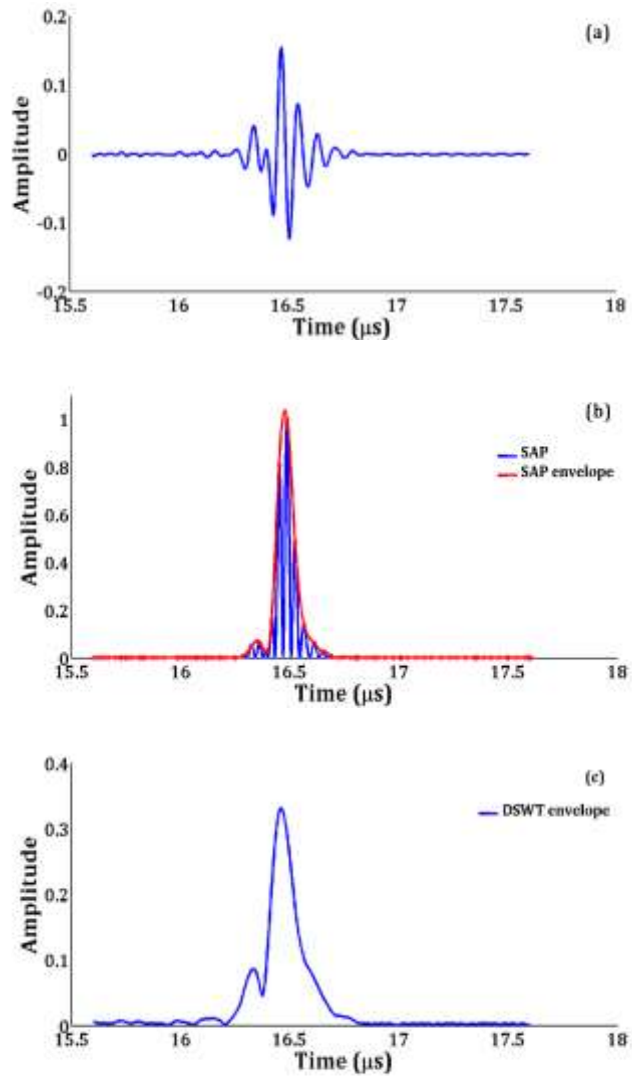

Figure 5: (a) Partially overlapping echoes backscattered from FSX 414 of 280 μm. (b)SAP approach envelope using db1wavelet for the scale 12. (c) DSWT envelope using db1wavelet for the level 4.

Table 4: Experiment results: SAP method.

Wavelet	TOF (μs)	Thickness (μm)	MSE (%)	Scale
db1	0.13	279.5	0.0017	12
sym1	0.13	279.5	0.0017	12. 15
db10, sym9	0.13	279.5	0.0017	8

corresponding reflected signals that exhibit partially and closely spaced overlapping echoes as shown in Figures 5a and 6a, respectively. The SAP and the DSWT methods have been applied and tested with several mother wavelets.

For the coating FSX 414 with a thickness of 280 μm, the estimated thickness using the SAP method was equal to 279.5 μm with MSE of 0.0017%. This result was obtained by db1 wavelet for the scale 12, sym1 wavelet for the scale 12 and 15, and db10, sym9 wavelet for the scale 8 as shown in Table 4. Figure 5b shows the SAP and its envelope obtained using db1 wavelet for the scale 12.

Table 5 shows the main results obtained by the use of the DSWT method. The best result was obtained by db1and sym1 wavelets for the level 4; and db2 and sym2 wavelets

Table 5: Experiment results: DSWT method.

Wavelet	TOF (µs)	Thickness (µm)	MSE (%)	Level
db1, sym1	0.13	279.5	0.0017	4
db2, sym2	0.13	279.5	0.0017	5

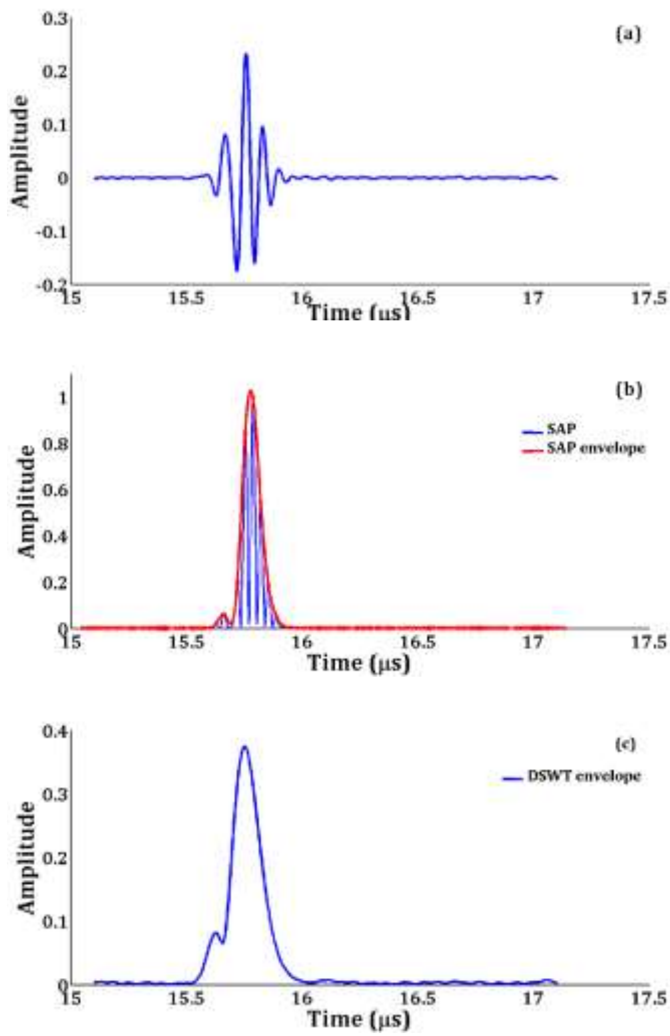

Figure 6: (a) Closely spaced overlapping echoes backscattered from FSX 414 of 180 µm. (b)SAP approach envelope using db7 wavelet for the scale 9. (c) DSWT envelope using db1wavelet for the level 4.

for the level 5. The estimated thickness and the MSE were 279.5 µm and 0.0017%, respectively. Figure 5c shows the DSWT envelope obtained using db1 wavelet for the level 4.

In the second case, when estimating the coating thickness of 180 µm, the obtained MSE by the use of both methods was more than 30%. The estimated thickness using the SAP method was 258 µm with MSE of 30.2%. This result was obtained using db7 wavelet for the scale 9 as shown in Figure 6b. Concerning the DSWT method, the estimated thickness was obtained using db1 wavelet for the level 4 as shown in Figure 6c. The estimated thickness and MSE were 258 µm and 30.2%, respectively.

From the obtained simulated results, it is possible to determine the mother wavelet and the appropriate decomposition level that best fits the real signals. Simulation results showed that in case of partially overlapping echoes, agreement was observed between simulation and experiments. Further, the good result had fallen on those obtained on simulation part. Concerning the closely spaced overlapping echoes, the real value of the TOF related to the coating thickness of 180 μm is approximately equal to 0.083 μs. Then, the TOF estimation is not consistent, because we have known already the accuracy and the limit of both methods as found previously in the simulation part for the TOF equal to 0.09 μs.

Conclusion

In the present study, two wavelet transforms on based methods namely, the SAP and DSWT were used to evaluate the TOF of ultrasonic signals. These methods were investigated by means of tests on both simulated and experimental signals taken from measured coating thickness. The numerical simulations provided the possibility to select the appropriate mother wavelets, as well as the appropriate level of decomposition to analyze partially and closely spaced overlapping echoes during an experimental test for measuring coating thicknesses. The accuracy and the limitations of these methods were highlighted.

The simulation results have shown that in the case of partially overlapping echoes, agreement was observed between simulation and experiments. The good result had fallen on those obtained in the simulation part.

Concerning the closely spaced overlapping echoes, the real value of the TOF related to the coating thickness of 180 μm is approximately 0.083 μs. In this case, the TOF estimation is not consistent because we have known already the limitations and accuracy of both methods as found previously in the simulation part for TOF equal to 0.09 μs.

These results showed good benefits of the wavelet methods for the time of flight estimation of ultrasonic signals. This suggests further study for defining a new mother wavelet that will be able to better TOF estimation, especially in critical conditions as closely spaced overlapping echoes.

Abbreviation: TOF, Time of flight; **SAP,** Scale-averaged power; **DSWT,** discrete stationary wavelet transform; **MSE,** mean square error; **NDT&E,** non-destructive testing and evaluation; **WT,** wavelet transform; **CWT,** continuous wavelet transform; **DWT,** discrete wavelet transform; **FWT,** fast wavelet transform; **MRA,** multi-resolution analysis; **SS,** stainless steel; **WGN,** white Gaussian noise ; **SNR,** signal to noise ratio

Nomenclature: $\mathbf{f_s}$, sampling rate; **N,** sampling points; $\boldsymbol{\psi_0}$, mother wavelet; $*$, conjugate; ***t*,** time; $\mathbf{y(t)}$, signal to be transformed; **s,** scale parameter; **τ,** translation parameter; **j,** level; $\mathbf{s_j}$, the scale at level j; **M,** the largest scale; $\boldsymbol{\varphi_{j,k}(t)}$, scaling function**;** $\boldsymbol{\psi_{j,k}(t)}$, wavelet function ; $\mathbf{j_0}$, coarsest scale; **j ,** dilatation; **k,** translation parameters; $\mathbf{a_{jk}}$, approximation coefficients; $\mathbf{d_{jk}}$, the detail coefficients; **x,** real signal; **ν,** white Gaussian noise (WGN); **β,** amplitude; **α,** bandwidth factor; **τ,** arrival time; $\mathbf{f_c}$, center frequency; ∅, phase; $\mathbf{z(t)}$, analytic signal; $\mathbf{\tilde{y}(t)}$, Hilbert transform; **i**, imaginary unit; $\boldsymbol{\theta(t)}$, phase.

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