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Research Paper

Some result on the harmonıc specıal serıes/analysıs

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ABSTRACT

The aim of this study is to get some new perspectives for divergence of some harmonic o special series such as alternating series, p-series, random series, fourier series, time series, and so on…Many different proofs of the divergences of such series are considered and proved. Also, several numerical illustrations suported our results.

Key words: Harmonic analysis, non- harmonic analysis, series, divergence.

INTRODUCTION

Analysis lectures are given especially in the first grades of the Faculty of Mathematics and Engineering of higher education. Their contents include both harmonic and nonharmonic series with their convergence or divergence. In generally, the first example in the topic is given as "$S = \sum_{n=1}^{\infty} \frac{1}{n}$" which is divergence but its general term is convergence.

Definition 1.1. A series whose terms are in harmonic progression as in $S = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ is called Harmonic Series.

The harmonic series are those whose terms contain the harmonic sum and diverge to infinity.

As it is known, Harmonic numbers have been at the center of attention of Mathematicians since the past. The series $\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}\}$ known as Harmonic Numbers or Harmonic Sequences is also used in many fields of Mathematics and arts. Besides, it is obvious that an instrument's timbre is uniquely determined with its harmonic series. Harmonic series are significant and influential in recognition whether or not are consonant.

As we know, there are numerous types of techniques for proving theorems or mathematical problems. In this study, our aim is to demonstrate the divergence of the Harmonic series using different methods and proofs. Comparisions of the different proofs in this study will be very significant and useful for readers and literature.

There are many different types of methods for convergence/divergences of series. These methods play an important bounds role for the $S = \sum_{n=1}^{\infty} \frac{1}{n^a} (a \in z^+)$ series. Partial series of the sums $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ converges to infinity very slowly, while $\frac{1}{n}$ converges to zero slowly.

Assume that $S = \sum_{n=1}^{\infty} \frac{1}{n}$ does not converges to infinity. It means that we use proof by contraposition. If we take/consider $n = 10^{12}$, we get following result by computing computer programme:

$$S_n = S_{10^{12}} = 1 + \frac{1}{2} + \cdots + \frac{1}{10^{12}}$$

This shows that it is less than 30. In a similar way, ıf we get $n = 10^{24}$, we find that S_n converges to positive integer number 60.

It has not been studied that the harmonic series may converge instead of divergence. Let's prove the divergence of this series with different evidence.

MAIN THEOREMS AND RESULTS

Divergence of the Harmonic Series

Our theorems and results are given as follows with the concepts of previous section.

Theorem 2.1. $S = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ is converges to infinity.

We give several different types of proofs as follows:

Proof 2.1. Let p be as follows:

$p = 1,2,3,\cdots,n$ and $p < x < p+1$

Then we have:

$$\frac{1}{p+1} < \frac{1}{x} < \frac{1}{p}$$

and

$$\int_p^{p+1} \frac{1}{p+1}\cdot dx < \int_p^{p+1} \frac{1}{x}\cdot dx < \int_p^{p+1} \frac{1}{p}\cdot dx$$
$$\Rightarrow \frac{1}{p+1}X < \ln x < \frac{1}{p}x$$
$$\Rightarrow \frac{1}{p+1}(p+1-p) < \ln(p+1) - \ln p < \frac{1}{p}(p+1$$
$$\Rightarrow \frac{1}{p} < \ln(p+1) - \ln p < \frac{1}{p} \ldots\ldots\ldots\ldots\ldots \mathbf{1}$$

We obtain n particle 1 in equations for $p = 1,2,3,\cdots,n$. If we write all of them as follows:

$$\frac{1}{2} < ln\,2 - ln\,1 < 1$$
$$\frac{1}{3} < ln\,3 - ln\,2 < \frac{1}{2}$$
$$\vdots$$
$$\frac{1}{n+1} < ln(n+1) - \ln n < \frac{1}{n}$$

then we get:

$$\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n+1} < ln(n+1) < 1+\frac{1}{2}+\cdots+\cdots \qquad 2$$

We also know that $f(x) = ln(x+1) \Rightarrow ln(x+1) < ln\,x + 1$ is satisfied. Ifwe put this inequation in the 2, we have:

$$ln(n+1) < S_n < ln\,n + 1$$

This proves that S_n converges to infinity. Besides, it gives that the divergency of the speed is $ln(n)$.

Proof 2.2. Let us suppose that $S = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\cdots$be a real number. It is trivial that the set of real numbers is a field. So, we can write S as the following:

$$S = 1+\frac{1}{2}+\frac{1}{3}+\cdots\cdot \qquad 1^*$$
$$S = \left(1+\frac{1}{3}+\frac{1}{5}+\cdots\right)+\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots\right)$$
$$S = \left(1+\frac{1}{3}+\frac{1}{5}+\cdots\right)+\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}\ldots\right)$$
$$S = \left(1+\frac{1}{3}+\frac{1}{5}\ldots\right)+\frac{1}{2}\cdot S$$
$$S-\frac{1}{2}S = \left(1+\frac{1}{3}+\cdots\right)$$
$$\frac{1}{2}S = (1+\frac{1}{3}+\cdots)\ldots\ldots\ldots \qquad 2^*$$

If the both left part of 1* and right part of 1* is divided by 2*, we have:

$$\frac{S}{2} = \frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots < 1+\frac{1}{3}+\frac{1}{5}\ldots = \frac{1}{2}S$$

This is a contradiction. Therefore,

$$S = 1+\frac{1}{2}+\frac{1}{3}+\cdots$$

is not finite sums.

Proof 2.3. As it is known that the following equation holds:

$$ln\,2 = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\ldots$$

We can consider such such as:

$$ln\,2 = (1-\frac{1}{2})+(\frac{1}{3}-\frac{1}{4})+(\frac{1}{5}-\frac{1}{6})+\cdots$$
$$= \frac{1}{2}+\frac{1}{12}+\frac{1}{30}+\cdots > 0\ldots\ldots \qquad 1^{**}$$

Now, assuming that $S = 1+\frac{1}{2}+\frac{1}{3}+\cdots$is convergent. In this case, we can rewrite $ln\,2$ by changing the ordering of the numbers. Hence, we have:

$$ln\,2 = (1+\frac{1}{3}+\frac{1}{5}+\cdots)-(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots)$$
$$= (1+\frac{1}{3}+\frac{1}{5}+\cdots)-\frac{1}{2}(1+\frac{1}{2}+\frac{1}{3}+\cdots)$$

Using Proof 2.2, we obtain:

$$ln\,2 = \frac{1}{2}H - \frac{1}{2}H = 0 \ldots\ldots \qquad 2^{**}$$

This is a contradiction and contradicts 1**. This indicates that $S = 1+\frac{1}{2}+\frac{1}{3}+\cdots$is divergent.

Proof 2.4. Supposing that $S = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n} = a \in \mathbb{R}$. It indicates that this sum is convergent. If we write a as follows:

$$a = \left(1+\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{1}{4}\right)+\cdots > \left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{6}+\frac{1}{6}\right)\ldots > a.$$

which is a contradiction. So, S is not convergent but divergent.

Proof 2.5. For $n \in Z^+$ and $n \neq 1$, we can use $\frac{2}{n} < \frac{1}{n-1} + \frac{1}{n+1}$ inequality. Then, we obtain:

$$\frac{2}{3} < \frac{1}{2} + \frac{1}{4}$$
$$\frac{2}{4} < \frac{1}{3} + \frac{1}{5}$$
$$\frac{2}{5} < \frac{1}{4} + \frac{1}{6}$$
$$\frac{2}{6} < \frac{1}{5} + \frac{1}{7}$$
$$\frac{2}{7} < \frac{1}{6} + \frac{1}{8}$$

for $n \in Z^+, n > 1$. Using them, we have:

$$2\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} \dots\right) < \frac{1}{3} + \frac{1}{4} + 2\left(\frac{1}{5} + \frac{1}{6} \dots\right)$$
$$2\left(\frac{1}{3} + \frac{1}{5}\right) < \frac{1}{3} + \frac{1}{4}$$

It is trival that this is a contradiction since 2a<a. Hence, S is divergent.

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