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Students' difficulties of understanding exponents and exponential expressions

ABSTRACT

The aim of this particular investigation was to find out persistent errors that students encounter when simplifying a given exponential expressions as well as to understand possible reasons such errors make. Fifty-six University students taking the course mathematics for geographer in Dilla University, Ethiopia were asked to simplify exponential expressions during their course assessment. A quantitative data analysis carried out on their work. Thus it was found that incomplete understanding of the concept of negative (-) was the root cause for most students' errors and misconceptions. In the present study, basic exponential rules were associated with negativity either as not properly perceived or wrongly interpreted by the students. Based on the result, it is recommended that students must develop basic concepts with understanding of additive inverse and multiplicative inverse to develop a more abstract understanding of negative operations in algebraic expression. Moreover, it is useful for students taking a similar course to understand laws of exponents involving negativity.

Key words: Exponentiation, negative numbers, concept image, additive inverse, multiplicative inverse.

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INTRODUCTION

Exponents and logarithms are important mathematical concepts that are important for modelling and understanding growth of population, compound interest and radioactive decay. Further, exponential and logarithmic functions have been considered as central concepts for many college and university courses of mathematics, including differential equations, calculus at large, and complex analysis. However, exponents and exponential expressions are important mathematical concepts, not much investigation has been done on students' learning difficulties and understanding of exponents. The need for studying exponents is to gain knowledge about secondary school students' mental constructions (Christou et al., 2007; Weber, 2002a, 2002b).

A study conducted by Vlassis (2002a, 2004) concluded that negative sign used for different purpose are counter intuitive and a difficulty for students after examining middle school students' conception of negativity and its interpretation in an expression. The study showed that students perceived the concept of negative as a process associated with binary operations of subtraction. A great deal of students' work in elementary algebra is controlled by the concepts of linear functions than exponential and power functions. Conceptualization of magnitude in a new way is required by learners during the transition to exponential functions (Kieran, 2007). Similar result has been documented by Barcellos (2005) on postsecondary students' improper use of the equal sign and the distributive law, and invalid cancellations when simplifying expressions. Student might write $2x - 4 = 6 + 4 = 10 = 5$ while solving $2x - 4 = 6$, in a test.

Cangelosi et al. (2013) found that negative sign errors

persist beyond other types of errors for students enrolled in College Algebra through Calculus II. In another instance, middle school students' misconceptions about the equals sign and negative sign have been found to be problematic for learning to solve algebraic equations (Booth and Koedinger, 2008).

Knuth et al. (2006) studied that equality and variable concepts' difficulties cause problems for learning algebra. The same problem is valid for the subject of equation, which has an important place in algebra. In fact, middle school students' perception about equality plays a critical role in students' equation solving and verbal problem solving achievement.

Students memorise algebraic rules without conceptual understanding attached to these concepts in a mathematics discourse. Many students have difficulty keeping track of and applying the rules appropriately; students often misinterpret -3^2 as equal $(-3)^2$ not $-(3)^2$. Some students interpret 3^{-2} as $3^{\frac{1}{2}}$ instead of $\frac{1}{3^2}$. The point is that an under-developed conception of additive and multiplicative inverse is at the root of these errors; and it is for that reason, among others, that the language used and the difficulty in interpreting notation and grouping may hinder students' progression (Cangelosi et al., 2013).

In most cases, many students memorize rules of exponential with no conceptual understanding or little and in their later stage, they may find it difficult to manipulating rules as well as using the rules correctly while moving from arithmetic (manipulation of numbers using basic operations eg. $2 - 7 = -5$) to algebra (study of algebraic expressions eg. $2x - 4 = 6$), mainly interpreting symbols in mathematics students is difficult (Carraher and Schliemann, 2007; Kieran , 2007).

According to Liston and O'Donoghue (2010), algebra gives the base for advancement of mathematical understanding and proficiency while working with problems involving algebraic expressions, which is necessary for those students who need to join mathematics, science, and technology education in their careers.

Even though some results are works from different kinds of students, the present study included 50 students attending the course mathematics for Geographer during the author's course delivery. The course involves the topic exponent and exponential expressions among others. It became clear that students had a fragile understanding of exponential expressions in general. It was observed that such error and understanding have some sort of similarity with lack of background knowledge. It became important to notify such problem to create remedy on the lower grade instruction. In this particular study, it was sought to identify students' errors and misconception made while simplifying exponential expressions and to see why these errors are made by students.

The study utilizes constructivist approach in which learners construct their knowledge about mathematical ideas from their own experiences. The process of learning needs the students to fit with the existing knowledge based on previous experiences to understand new knowledge. So that the study utilized constructivist framework, which gives priorities on, how students construct their own knowledge with correct conception. In this case, development of students' knowledge on construction in relation to exponential expressions in necessary (Sfard, 1991, 1992). Such a framework is developed on notions of *concept definition* and *concept image* (Tall and Vinner, 1981).

Contribution to literature

-The main purpose of conducting this study was to identify persistent errors that students make when working with exponential expressions to help teachers to reconsider their methodology in the way that can help student to develop correct understanding.
-This study was conducted on non-mathematics learner (geography students), but similar problem and misconception could be seen for other students learning and working with exponential expression.

Objective

The major objective of this study was to determine the possible error and misconceptions of students while working with and simplifying exponential expressions, and to understand why students make these errors.

METHODS

The purpose of this study was to identify persistent errors that students make when working with exponential expressions. To accomplish the objective, an assessment was administered to 50 University students taking the course mathematics for geographer in Dilla University in degree program. Since the author was their instructor for this course, it was good opportunity to get data as part of course, assessment. Quantitative research approach was preferred to analyse coded students score in test items while identifying their error and difficulties.

The test consisted three sections: Section I includes eight items, asked to simplify exponential expressions; Section II, asked to compare (relative magnitudes) six pairs of exponential expressions using <, > or = (the relational

symbols) and Section III, they were requested to determine the sign (positive or negative) of the given exponential expression. All assessment questions for section I, II and III were drawn from course mathematics for Geographer taught by the author.

Data collection tool (students' assessment) is shown hereafter.

Assessment /data collection tool

I. Simplify the expression below and express your answer using *positive exponents:*

Ia. $(16x^3)^2$ Ib. $(-8)^{2/3}$ Ic. $\sqrt[3]{27^2}$ Id. $-9^{3/2}$ Ie. $27^{(-2/3)}$
If. $(-4)^{3/2}$ Ig. $8^{(-1/3)}$ Ih. $(\frac{9}{4})^{3/2}$ Ii. $5^0 - 5^{-1}$ Ij. $(\frac{9}{4})^{-2}$

II. Compare the following expression using the symbols < , = or >

IIa. -15^8_____$(-15)^8$ IId. -17^8_____$(17)^{-8}$
IIb. $(-12)^{-8}$_____$(-12)^{-14}$ IIe. $23^{3/5}$_____$15^{3/5}$
IIc. 0.5^{25}_____0.5^{33} IIf. $12^{5/3}$_____$\sqrt[3]{12^5}$

III. Label each of the following numbers as only either positive or negative:

IIIa. $(-3)^{12}$_____IIIc. $\left(-\frac{1}{2}\right)^{-4}$ _____

IIIb. $\left(-\frac{1}{3}\right)^{-1/3}$ _____IIId. 2^{-3}_____

RESULTS

The results are presented in Tables 1, 2 and 3. Students' examination score with correctly and not correctly answered is shown in Table 1.

The table shows that students' correct response varies from item to item. Section I asked students to simplify the expression and express their final answer using *positive exponents.*

Only for g, h and j questions, that majority of students (75, 66.07 and 66.07%, respectively) successfully answer the question by expressing their result using *positive exponents.*

Namely $8^{(-1/3)} = 1/2, (\frac{9}{4})^{3/2} = 27/8$ and $(\frac{9}{4})^{-2} = 2/3$, on the other hand 89.29% of students has got difficulty in simplifying, $5^0 - 5^{-1}$ item i, sample students work indicated that while simplifying $5^0 - 5^{-1}$, they treated $5^0 = 0$, this error may be misconception of the rule $x^0 = 1$, for all x different from 0. One-sample students worked as

$5^0 - 5^{-1} = 0 - \frac{1}{5} = -1/5$ and unable to convert to positive expression.

Most students see item c $\sqrt[3]{27^2}$ as difficult to change radical to exponent $\sqrt[m]{x^n} = x^{n/m}$, no correct response was given by most students in terms of exchanging of m and n error, such as $\sqrt[m]{x^n} = x^{m/n}$.

One student who arrived at correct answer using incorrect notation, by considering () presence and absent treated as $\sqrt[3]{-8^2} = \sqrt[3]{64} = 4$.

It appears that a student treated -8^2, as $(-8)^2$, actually they are different. Many other students who are incorrect in simplification, their error and misconception arise from moving the negative sign inappropriately. Thus a student worked as $(-8)^{2/3} = -8^{2/3} = -(8^{2/3}) = -4$. The other one moved the negative sign to exponent $(-8)^{2/3} = 8^{-2/3} = 1/4$.

The overall performance on simplifying and expressing result using positive exponent is shown in Figure 1.

Section I, asked them for simplifying the given exponential expressions. They, in general, expressed this section to be the most difficult of the three, since it involved practical work.

While comparing -15^8 and $(-15)^8$ IIa, only 16.07% (9) students correctly answered this question, almost 83.93% were not correct, among these students almost 50% said equal "=" this students did not recognized the role of bracket () in dealing with Exponent. $-15^8 = -(15^8)$, is negative but $(-15)^8 = (-1)^8(15)^8 = (15)^8$ is positive. Therefore, majority of the students did not properly understand the role of bracket while dealing with exponents.

In question IIb, almost 80.36% of students failed to properly compare. These students $(-12)^{-8}$ was smaller than $(-12)^{-14}$. This may be due to their impression that expressions are negative, and not recognized as even exponents makes power positive even though the base is negative number.

Here also, students are not successful in giving discussion on expression as positive or negative. Among others in item c, only 12.5% of students labelled $\left(-\frac{1}{2}\right)^{-4}$ as positive. Students here also have misconception about negative sign, and how it can be treated. 87.5% students perceived $-\frac{1}{2}$ considered to be used to decide the overall sign of the power without consideration, or have misconception on () and -4 as even exponent. Overall performance was still below 50% as only 28 (49.11%) gave correct response.

DISCUSSION

From the result, possible error was committed relating to

Table 1: Students achievement score for Section I.

Section I question	Correct		Not Correct		Total
	N	%	n	%	
A	10	17.86	46	82.14	56
B	9	16.07	47	83.93	56
C	9	16.07	47	83.93	56
D	13	23.21	43	76.79	56
E	9	16.07	47	83.93	56
F	5	8.93	51	91.07	56
G	42	75.00	14	25.00	56
H	37	66.07	19	33.93	56
I	6	10.71	50	89.29	56
J	37	66.07	19	33.93	56
Overall score	18	31.61	38	68.39	56

Table 2: Compare expression using <> or =.

Section II question	Correct		Not correct		Total
	n	%	n	%	
-15^8_____$(-15)^8$	9	16.07	47	83.93	56
$(-12)^{-8}$_____$(-12)^{-14}$	11	19.64	45	80.36	56
0.5^{25}_____0.5^{33}	21	37.50	35	62.50	56
-17^8_____$(17)^{-8}$	44	78.57	12	21.43	56
$23^{3/5}$_____$15^{3/5}$	14	25.00	42	75.00	56
$12^{5/3}$_____$\sqrt[3]{12^5}$	17	30.36	39	69.64	56
Overall	26	45.83	30	54.17	56

Table 3: Students achievement on labelling exponents either positive or negative.

Section III question	Correct		Not Correct		Total
	n	%	n	%	
A	37	66.07	19	33.93	56
B	27	48.21	29	51.79	56
C	7	12.50	49	87.50	56
D	39	69.64	17	30.36	56
Overall	28	49.11	29	50.89	56

negative sign in exponential expression. **Sticky sign (-)** is one category of error where students wrongly include the negative sign as part of base. Example of such errors is misinterpretation of a student, $-9^{3/2} = (-9)^{3/2}$ considered as equal. For the negative sign here, the student interpreted it as "stuck" to 9 instead of realizing that $-9^{3/2}$ is the additive inverse of $9^{3/2}$, like -2 is the additive inverse of 2 in integer. Other category may be difficulty in which students either inappropriately move negative sign or "flip" a number as

the roaming reciprocal within the expression. We can see one instance for this $2^{-3} = 2^{1/3}$ or $2^{-3} = -2^3$ or $2^{-3} = 3/2$. Errors here may be related to immature conceptions of inverse. For this, it can be mentioned as use of **language, notation and grouping** as contributing factors for the error and misconception.

On the other hand, there is need for prerequisite knowledge of algebra. This is because these students are from social science background and as such, have little

Figure 1: Section I overall achievement.

background knowledge about exponential and logarithm function. In my teaching, I have also noticed that students get difficulties adding subtraction with negative numbers.

Grouping, while considering their difficulties interpreting $-9^{3/2}$ as $(-9)^{3/2}$. Students considered -9 as a single object, which is not separable and then raised to the $\frac{3}{2}$ power. Such kind of error of students can be categorized as grouping error.

Learners just considered the negative sign attached to the number 9, rather than understanding it as unary operation of negation, or taking it as additive inverse for $9^{3/2}$, not just additive inverse of nine(9). One possible explanation for this is that students often believe () parentheses do not matter.

It can be mentioned the error reasoning seen in two students' work $(-4)^{3/2} = -4^{3/2} = -(4)^{3/2} = -9$ and $(-8)^{2/3} = -8^{2/3} = -(8)^{2/3} = -4$. These students do not perceived the role of grouping mentioned by the parentheses. Such errors show that they are at stage of internalization where similarity and difference between concepts $-m = (-m$ and $-m^2 \neq (-m)^2)$ not yet understood by students (Sfard, 1991). There were many instances observed on students' work that magnifies errors related to use of negativity in power expression. It has also been noted that the conception of negativity here by students in the case of subtraction could be developed in relation to additive inverse. Such concept formation is in fact from early background stage of development. Moreover, it was observed that they lack reflecting multiplicative inverse.

Notably, the study observed students arriving correct answer by wrong procedure and rule of exponents. It may be learners mentally working correctly but writing incorrectly not giving attention to procedures. One example

of such kind is $(-8)^{2/3} = \sqrt[3]{-8^2} = \sqrt[3]{64} = 4$. Here they did not write $(-8)^{2/3} = \sqrt[3]{(-8)^2}$ these students appeared to recognize that -8 was the quantity to squire. Such way of individual working style is not seen to hinder their ability and skill in simplifying the given expression; instead used to keep of their mental image or process. It was observed in similar research as learners usually could work by their own way to arrive at correct solution but in a wrong way (Barcellos, 2005). Most common example is exchangeable use of $=$ and $\Rightarrow$, like $\frac{2}{x} = \frac{1}{5} = x = 10$ rather than writing $\frac{2}{x} = \frac{1}{5} \Rightarrow x = 10$.

Conclusions

The aim of this study was to find out error and possible misconception of students in utilizing exponential expression in mathematics for geography course. The study identified errors and misconceptions in different way, one that sticky sign, grouping, roaming reciprocals. Two main concerns are raised here as remedy or implication from this result for correcting the problem: first there should be matured development of mathematical conceptions starting from the beginning of introduction particular knowledge and the other is that the language we use while developing a particular mathematics concept has an influence on correct construction of knowledge .

An operational level of mathematics concept construction such as that of equal sign interacted with students' mathematical development. Moreover, operational understanding alone interferes with students' mathematical development unless it changed to structural understanding (Knuth et al., 2006; Weber, 2002).

Rational powers would be an obstacle for students' reasoning while working problems involving exponential

expression (Christou et al., 2007). In this study, it was found that learners understanding and reasoning of additive and multiplicative inverses prevented them in some instances to understand properly.

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